Branching-Selection Particle Systems

James Stephens

Stanford Mathematics

Branching-Selection Systems

We consider systems of a finite (possibly growing) population of particles on $\mathbb{R}$.

- Particles diffuse and drift.
- At random times, particles branch.
- At random times (possibly dependent on branch events), particles are deleted.

Question: what macroscopic speed/profile is selected as the number of particles goes to infinity? As time goes to infinity? Can we use this information to prove properties of the microscopic system?

Related to conjectures of Brunet and Derrida, mathematical biology, nonlinear PDEs.

Example: The Fisher-KPP equation

N Brownian particles move on $\mathbb{R}$. Each particle carries a rate 1 clock. When the clock rings, the particle selects another uniformly at random. If the chosen particle is to the right of the ringing particle, the ringing particle jumps on top of the chosen particle.

Theorem (Groisman-Jonckheere-Martínez 2019)

With

$$u^N(t,x) := \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_{\{X_i(t)>x\}},$$

as $N \to \infty$*,*

$$u^N(t,x) \longrightarrow u(t,x),$$

where u *solves the Fisher-KPP equation*

$$\partial_t u = \frac{1}{2}\partial_{xx} u + u(1-u).$$

Selection Principles

In the previous setting, Groisman, Jonckheere, and Martínez show (using the hydrodynamic limit) that

$$\lim_{t \to \infty} \frac{X_{(1)}}{t} = \lim_{t \to \infty} \frac{X_{(N)}}{t} = v_N$$

and $\lim_{N \to \infty} v_N = \sqrt{2}$ where $\sqrt{2}$ is the minimum speed of a traveling wave solution to the Fisher-KPP equation. We call this a **weak selection principle**.

Using the strategy of Julien Berestycki and Oliver Tough, we can show a **strong selection principle**: the empirical tail under stationarity (centered at the median particle) converges in shape to the minimum speed traveling wave.

- Show compactness (of stationary distributions).
- Exhibit subsequential limits as weak solutions of PDEs.
- Use uniqueness of traveling wave solutions to conclude.